K. S. K COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF SCIENCE AND HUMANITIES

IMPORTANT QUESTIONS

(PART A & B)

SUBJECT CODE **:** MA6351 **YEAR/ SEM** **:** II / III

SUBJECT NAME **:** TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

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Sub.Code : MA6351 **Branch: Common to All**

Staff Name : Ms.K.GAYATHRI **Year / SEM: II/III**

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OBJECTIVES:

- To introduce Fourier series analysis which is central to many applications in engineering apart from its use in solving boundary value problems.
- To acquaint the student with Fourier transform techniques used in wide variety of situations.
- To introduce the effective mathematical tools for the solutions of partial differential equations that model several physical processes and to develop Z transform techniques for discrete time systems.

UNIT I PARTIAL DIFFERENTIAL EQUATIONS 9+3

Formation of partial differential equations – Singular integrals -- Solutions of standard types of first order partial differential equations - Lagrange's linear equation -- Linear partial differential equations of second and higher order with constant coefficients of both homogeneous and non-homogeneous types.

UNIT II FOURIER SERIES 9+3

Dirichlet's conditions – General Fourier series – Odd and even functions – Half range sine series – Half range cosine series – Complex form of Fourier series – Parseval's identity – Harmonic analysis.

UNIT III APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS 9+3

Classification of PDE – Method of separation of variables - Solutions of one dimensional wave equation – One dimensional equation of heat conduction – Steady state solution of two dimensional equation of heat conduction (excluding insulated edges).

UNIT IV FOURIER TRANSFORMS 9+3

Statement of Fourier integral theorem – Fourier transform pair – Fourier sin Four cosine transforms – Properties – Transforms of simple functions – Convolution theorem – Parseval's identity.

UNIT V Z - TRANSFORMS AND DIFFERENCE EQUATIONS 9+3

Z- transforms - Elementary properties – Inverse Z - transform (using partial fraction and residues) – Convolution theorem - Formation of difference equations – Solution of difference equations using Z - transform.

TOTAL (L:45+T:15): 60 PERIODS

UNIT I

PARTIAL DIFFERENTIAL EQUATIONS

PART A

1. Form the PDE from $(x-a)^2 + (y-b)^2 + z^2 = r^2$
2. Find the Complete integral of $p + q = pq$
3. Form the PDE by eliminating the arbitrary function from $z^2 - xy = f(\frac{x}{z})$
4. Solve $(D^2 - 7DD' + 6D'^2)z = 0$
5. Form the PDE by eliminating the arbitrary constants $\boldsymbol{a}$and $\boldsymbol{b}$from $z = (x^2 + a)(y^2 + b)$
6. Solve the equation $(D - D')^3 z = 0$
7. Eliminate the arbitrary function $'f'$ from $z = f(\frac{y}{x})$ and form the PDE
8. Solve $(D-1)(D - D' + 1)z = 0$
9. Find the PDE of the family of spheres having their centre on the $'z'$ axis
10. Solve $(D^4 - D'^4)z = 0$
11. Form the PDE by eliminating the arbitrary constants $\boldsymbol{a}$,$\boldsymbol{b}$from the relation $z = ax^3 + by^3$
12. Find the Particular integral of $(D^2 - 2DD' + D'^2)z = e^{x-y}$
13. Solve $(D^3 - 2D^2D')z = 0$
14. Find the PDE of all planes cutting equal intercepts from the x and y axis.
15. Find the complete integral of $q = 2px$
16. Form the PDE by eliminating the arbitrary constant from $z = ax + by + ab$
17. Form the PDE by eliminating the arbitrary constant from $z = (2x^2 + a)(3y - b)$
18. Form the PDE by eliminating the arbitrary function from $z = f(xy)$
19. Solve $\sqrt{p} + \sqrt{q} = 1$
20. Find the P.I of $(D^2 - 3DD' + 2D'^2)Z = \cos(x + 2y)$

PART B

1. (i) Form the PDE by eliminating the arbitrary function ϕ from
$\phi(x^2 + y^2 + z^2, ax + by + cz) = 0$ (8)
(ii) Solve $(D^2 + 2DD' + D'^2)z = \sinh(x+y) + e^{x+2y} + 4$ (8)
2. (i) Solve the partial differential equation $(mz - ny)p + (nx - lz)q = ly - mx$ (8)
(ii) Solve the equation $(D^3 + D^2D' - 4DD'^2 - 4D'^3)z = \cos(2x + y)$ (8)
3. (i) Solve $(D^2 + 3DD' - 4D'^2)z = \cos(2x + y) + xy$ (8)
(ii) Solve $z^2(p^2 + q^2) = (x^2 + y^2)$ (8)
4. (i) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$ (8)
(ii) Solve $(D^2 - DD' + 2D)z = e^{2x+y} + 4$ (8)

5. (i) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$ (8)

(ii) Solve $(y-xz)p+(yz-x)q=(x+y)(x-y)$ (8)

6. (i) Solve $(D^3-7DD'^2-6D'^3)z=\sin(2x+y)$ (8)

(ii) Solve $z=px+qy+p^2q^2$ (8)

7. (i) Solve $(x-2z)p+(2z-y)q=y-x$ (8)

(ii) Solve $(D^2-D'^2)z=e^{x-y}\sin(2x+3y)$ (8)

8. (i) Find the Singular integral of $z=px+qy+p^2+pq+q^2$ (8)

(ii) Solve $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$ (8)

9. (i) Solve $(2D^2-DD'-D'^2+6D+3D')z=xe^y$ (8)

(ii) Solve the partial differential equation $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$ (8)

10. (i) Solve $x^2p^2+y^2q^2=z^2$ (8)

(ii) Form the PDE by eliminating the arbitrary function from the relation $z=f(x+ct)+g(x-ct)$ (8)

UNIT II

FOURIER SERIES

PART A

1. Find the co-efficient b_n of the Fourier series for the function $f(x)=x\sin x$ in (-2,2).
2. Find the Root Mean Square value of the function $f(x)$ over the interval (a,b)
3. Find the constant term in the expansion of $f(x)=\cos^2 x$ as a Fourier series in the interval $(-\pi,\pi)$.
4. State the Dirichlet's conditions for Fourier series.
5. What is meant by Harmonic Analysis?.
6. State the sufficient condition for a function $f(x)$ to be expressed as a Fourier series
7. Obtain the first term of the Fourier series for the function $f(x)=x^2,(-\pi,\pi)$
8. Find the RMS value of $f(x)=x^2$ in $(0,l)$.
9. Find the Root Mean Square value of $f(x)=x$ in $(0,l)$.
10. If the Fourier series corresponding to $f(x)=x$ in $(0,2\pi)$ is $\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{l}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{l}$ Without finding the values of a_0, a_n, b_n. Find the value of $\frac{{a_0}^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}({a_n}^2+{b_n}^2)$
11. Does $f(x)=\tan x$ possess a Fourier expansion.
12. If $f(x)$ is discontinuous at $=a$, what does its Fourier series represent at that point?
13. Determine b_n in the Fourier series expansion of $f(x)=\frac{\pi-x}{2}$ in $0<x<2\pi$ with period 2π.
14. If $f(x)=2x$ in the interval (0,4), then find the value of a_2 in the Fourier series expansion.

15. What are the constant term a_0 and the coefficient of $\cos nx$, a_n in the Fourier series expansion of $f(x) = x - x^3$ in $(-\pi, \pi)$?

16. Determine the value of a_n in the Fourier series expansion $f(x) = x^3$ in $(-\pi, \pi)$.

18. Find b_n in the expansion of $f(x) = \begin{cases} 1 + \frac{2x}{\pi} ; -\pi < x < 0 \\ 1 - \frac{2x}{\pi} ; 0 < x < \pi \end{cases}$ in $(-\pi, \pi)$.

19. Find Fourier sine series for $f(x) = 1, \quad 0 < x < \pi$.

20. State Parseval's identity for full range expansion of $f(x)$ as Fourier series in $(0,2l)$.

PART B

1. (i)Expand $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, 0 < x < \pi \end{cases}$ as a full range Fourier series in the interval $(-\pi, \pi)$.

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \infty = \frac{\pi^2}{8}$. (8)

(ii) Find the half-range sine series of $f(x) = 4x - x^2$ in the interval (0,4). Hence deduce the value of the series$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots .. \infty$ (8)

2. (i) Obtain the sine series for $f(x) = \begin{cases} x \ in \ 0 \le x \le \frac{l}{2} \\ l - x \ in \frac{l}{2} \le x \le l \end{cases}$ (8)

(ii) Find the Fourier series up to second harmonic for $y = f(x)$ from the following values. (8)

x:	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y:	1.0	1.4	1.9	1.7	1.5	1.2	1.0

3.(i)Expand $f(x) = x(2\pi - x)$ as Fourier series in $(0,2\pi)$ and hence deduce that the sum of (8)
$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots.$

(ii) Obtain the Fourier series for the function $f(x)$ given by $f(x) = \begin{cases} 1 - x, -\pi < x < 0 \\ 1 + x, 0 < x < \pi \end{cases}$. Hence (8)
deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \infty = \frac{\pi^2}{8}$.

4. (i) Find the Fourier series of x^2 in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots . = \frac{\pi^4}{90}$ (10)

(ii) Obtain the Fourier cosine series of $f(x) = \begin{cases} kx, 0 < x < \frac{l}{2} \\ k(l - x), \frac{l}{2} < x < l \end{cases}$ (6)

5. (i) Obtain the Fourier series of periodicity 3 for $f(x) = 2x - x^2 \ in \ 0 < x < 3$. (6)
(ii) Obtain the Fourier series of $f(x) = x sinx \ in \ (-\pi, \pi)$ (10)

6. (i)Find the Fourier series of $f(x) = (\pi - x)^2 \ in \ (0,2\pi) of \ periodicity \ 2\pi$. (4)

(ii) Calculate the first 3 harmonics of the Fourier of $f(x)$ from the following data (12)

X	0	30	60	90	120	150	180	210	240	270	300	330
Y	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2.0

7.(i) Obtain the Fourier series of the periodic function defined by $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$.

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots + \infty = \frac{\pi^2}{8}$. (8)

(ii) Compute upto first harmonics of the Fourier series of $f(x)$ given by the following table. (8)

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
f(x)	1.98	1.30	1.05	1.30	-0.38	-0.25	1.98

8.(i) Expand $f(x) = x - x^2$ as a Fourier Series in $-L < x < L$ and using this series find the root mean square value of $f(x)$in the interval. (10)

(ii) Find the complex form of the Fourier series of $f(x) = e^x$in $-1 < x < 1$. (6)

9.(i) Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$. (4)

(ii) Find the Fourier series as far as the second harmonic to represent the function $f(x)$with the period 6, given in the following table (12)

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

10.(i) Find the Fourier series expansion of $f(x) = x + x^2$ in ($-\pi, \pi$). (8)

(ii) Find the Fourier series expansion of $f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x \le 2 \end{cases}$. (8)

Also deduce $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ to $\infty = \frac{\pi^2}{8}$

UNIT III

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART A

1. The ends A and B of a rod of length 10 cm long have their temperature kept at 20^0C and 70^0 C. Find the steady state temperature distribution on the rod.
2. Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is f(x) and the initial velocity imparted at each point x is g(x).

3. In steady state conditions derive the solution of one dimensional heat flow equation.
4. What is the basic difference between the solutions of the one dimensional wave equation and one dimensional heat equation.
5. What are the possible solutions of one dimensional wave equation?
6. What are the possible solutions of one dimensional heat equations?
7. Write down the possible solutions of the Laplace equation.
8. In wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ what does a^2 stands for?
9. State any two laws which are assumed to derive one dimensional heat equation.
10. A rod 30 cm long has its ends A and B kept at 20^0C and 80^0C respectively until steady state conditions prevail. Find the steady state temperature in the rod.
11. An insulated rod of length l=60 cm has its ends a and B maintained at 30^0 C and 40^0C respectively. Find the steady state solutions.
12. Write any two solutions of the Laplace equation obtained by the method of separation of variables.
13. How many boundary conditions are required to solve $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$?
14. State one dimensional heat equation with the initial and boundary conditions.
15. Distinguish between steady and unsteady states in heat conduction problems.
16. What is the steady state heat equation in two dimensions in Cartesian form?
17. Write down Laplace equation in Cartesian coordinates.
18. What are the conditions assumed in deriving the one dimensional Wave equation.

PART B

1. A string is stretched and fastened to two points x = 0 and x= l apart. Motion is started by displacing the string into the form y = k (l x – x^2) from which it is released at time t=0. Find the displacement of any point on the sting at a distance of x from one end at time t. (16)
2. A string of length 2l is fastend at both ends. The mid point of the string is taken to a height b and then released from in that position. Show that the displacement is $y(x,t) = \frac{8b}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} \sin\left(\frac{(2n-1)\pi x}{2l}\right) \cos\left(\frac{(2n-1)\pi a t}{2l}\right)$ (16)

3. A tightly stretched string with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If it is set vibrating string giving each point a velocity $\lambda x(l-x)$, show that displacement $y(x,t) = \frac{8\lambda l^3}{a\pi^4}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}\sin\left(\frac{(2n-1)\pi x}{l}\right)\sin\left(\frac{(2n-1)\pi a t}{l}\right)$ (16)
4. A tightly stretched string with fixed end points x=0 and x=L is initially at rest in its equilibrium position. If it is set vibrating string giving each point a velocity 3x(L-x), find the displacement. (16)
5. If a string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $v_0 \sin^3\left(\frac{\pi x}{l}\right)$, $0 < x < l$, determine the displacement of a point distant x from one end at time t. (16)
6. A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity given by $v = \begin{cases} cx, & in\ 0 \le x \le \frac{l}{2} \\ (l-x), & in\ \frac{l}{2} \le x \le l \end{cases}$ Find the displacement function y(x, t). (16)
7. A rod 30 cm long has its ends A and B kept at 20^0C and 80^0C respectively until steady state conditions prevail the temperature at each end is then suddenly reduced to 0^0 c and kept so. Find the resulting temperature function u(x,t) taking x=0 at A. (16)
8. A bar 10 cm long with insulated sides, has its ends A and B kept at 20^0 C and 40^0C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50^0C and at the same time instant that at B is lowered to 10^0C and maintained there after. Find the subsequent temp. distribution in the bar. (16)
9. The ends A and B of a rod l cm having the temperatures 40^0C and 90^0C respectively until steady state conditions prevails. The temperature at A is then suddenly raised to 90^0C and at the same time instant that at B is lowered to 40^0C and maintained there after. Find the temp. distribution of the rod at a time t. Also show that the temperature at the mid point of the rod remains unaltered for all time, regardless of the material of the rod. (16)

10. A rod of length l has its ends A and B kept at 0^0 C and 100^0C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 25^0C and at the same time instant that at B is lowered to 75^0C.
    Find the temperature u(x,t) at a distance x from A at time t. (16)
11. A rod of length l has its ends A and B kept at3 0^0 C and 80^0C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 40^0C and at the same time instant that at B is lowered to 60^0C.
    Find the temperature distribution of the rod after time t. (16)
12. Find the steady state temp. distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions u(0,y) = u(a,y) = 0 for $0 \le y \le b$ u(x,b) = 0 and u(x,0) = x(a-x) for $0 \le x \le a$. (16)
13. The boundary value problem governing the steady state temperature distribution in a flat , thin, square plate is given by $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$**, 0 < x < a, 0 <y < a** u(x,0) = 0,
    u(x,a) = 4 sin $^3\left(\frac{\pi x}{a}\right)$, **0 < x < a ,** u(0,y) = 0 , u(a,y) = 0 ,**0 <y < a**
    find the steady state temperature distribution in the plate. (16)
14. A rectangular plate is bounded by the lines x= 0, y = 0, x = a and y = b. Its surfaces are insulated and the temperature along two adjacent edges are kept at 100^0C, while the temperature at any point along the other edges are at 0^0 C. Find the steady state temperature at any point in the plate. Also find the steady state temperature at any point of a square plate of side 'a' if two adjacent edges are kept at 100^0 C and the others at 0^0 C. (16)
15. An infinitely long rectangular plate with insulated surface is 10 cm wide. The two long edges and one short edge are kept at zero temperature given by $u = \begin{cases} 10y, & for \quad 0 \le y \le 5 \\ 20(10-y), & for \quad 5 \le y \le 10 \end{cases}$.
    Find the steady state temperature distribution in the plate. (16)

UNIT IV

FOURIER TRANSFORMS

PART A

1. State the Fourier integral theorem.
2. Write the Fourier transform pair.
3. Find the Fourier transform of $f(x)=\begin{cases}1 & in\ |x|<a\\ 0 & in\ |x|>a\end{cases}$
4. Define infinite Fourier transform write inverse formula also.
5. Define Fourier sine transform and its inverse.
6. Find the Fourier sine transform of $f(x)=e^{-ax}$ (a > 0).
7. Find the Fourier sine transform of $f(x)=e^{-x}$.
8. Find Fourier sine transform of $\frac{1}{x}$
9. Find the Fourier sine transform of $f(x)=e^{ax}$
10. Define Fourier cosine transform and its inverse.
11. Find Fourier cosine transform of $f(x)=e^{-x}$
12. If the Fourier transform of $f(x)$ is F(s) then prove that . $F[f(x-a)]=e^{isa}F(s)$
13. State the Fourier transforms of the derivatives of a function.
14. Prove that $F[f(ax)]=\frac{1}{a}F\left(\frac{s}{a}\right)$, $a>0$
15. If F(s) is the Fourier transform of $f(x)$ then $F[f(x)\cos ax]=\frac{1}{2}[F(s+a)+F(s-a)]$
16. If F(s) is the Fourier transform of $f(x)$ then prove that $F[x.f(x)]=(-i)\frac{dF(s)}{ds}$
17. If F(s) is the Fourier transform of $f(x)$ then $F[f(ax)]=\frac{1}{a}F\left(\frac{s}{a}\right)$, $a>0$
18. If F(s) is the Fourier transform of $f(x)$ then $F_S[f(x)\cos ax]=\frac{1}{2}[F_S(s+a)+F_S(s-a)]$
19. Prove that $F_C[f(ax)]=\frac{1}{a}F_C\left(\frac{s}{a}\right)$, $a>0$
20. State the convolution theorem of the Fourier transform.

PART B

1. (i) Find the Fourier cosine transform of $e^{-a^2x^2}$ (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using transforms (8)

2.(i) Find the Fourier sine transform of $f(x)=\begin{cases}\sin x, & 0<x\le a\\ 0, & a\le x<\infty\end{cases}$ (8)

(ii) Find the Fourier Transform of $f(x)=\begin{cases}x & in\ |x|\le 1\\ 0 & in\ |x|>1\end{cases}$ (8)

3. Find the Fourier Transform of $f(x)=\begin{cases}1-x^2 & in\ |x|\le 1\\ 0 & in\ |x|>1\end{cases}$. Hence prove that (16)

(i) $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$.

(ii) $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds=\frac{\pi}{15}$.

4.(i) Find the Fourier transform of f(x) if $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a>0\end{cases}$.Hence deduce that $\int_0^\infty\left(\frac{\sin t}{t}\right)dt=\frac{\pi}{2}$

and $\int_0^\infty\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$ (10)

(ii)Find the Fourier sine transform of $f(x)=\begin{cases}x, & 0<x<1\\ 2-x, & 1<x<2\\ 0 & x>2\end{cases}$ (6)

5. Find the Fourier transform of f(x) if $f(x)=\begin{cases}1, & |x|<2\\ 0, & |x|>2\end{cases}$.Hence deduce that (i) $\int_0^\infty\left(\frac{\sin x}{x}\right)dx$

(ii) $\int_0^\infty\left(\frac{\sin x}{x}\right)^2 dx$ (16)

6.(i) Find the Fourier Transform of $f(x)$ if $f(x)=\begin{cases}1-|x|, & |x|<1\\ 0, & |x|>1\end{cases}$. Hence deduce that

$\int_0^\infty\left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$ (10)

(ii) Derive the Parseval's identity for Fourier transforms. (6)

7. Show that the Fourier transform of $f(x)=\begin{cases} a^2-x^2 & |x|<a \\ 0 & |x|>a>0 \end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - as\cos as}{s^3}\right)$.

Hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}\cos\frac{t}{2}\,dt=\frac{\pi}{4}$. Using Parseval's identity show that

$\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt=\frac{\pi}{15}$. (16)

8.(i) Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases} 1, & |x|<1 \\ 0, & otherwise \end{cases}$ Hence deduce that

$\int_0^\infty \frac{\sin x}{x}dx=\int_0^\infty \frac{\sin^2 x}{x^2}dx=\frac{\pi}{2}$ (8)

(ii) Using Parseval's identity calculate $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$, (8)

9.(i) Using Parseval's identity calculate (ii) $\int_0^\infty \frac{x^2}{(a^2+x^2)^2}dx$ if a > 0. (8)

(ii) Find the Fourier transform of e^{-4x} and hence deduce that $\int_0^\infty \frac{\cos 2x}{x^2+16}dx=\frac{\pi}{8}e^{-8}$ (8)

10. (i)Find the Fourier transform of e^{-ax} and hence deduce that $\int_0^\infty \frac{x\sin 2x}{x^2+16}dx=\frac{\pi}{2}e^{-8}$ (8)

(ii) State and prove convolution theorem for Fourier transforms. (8)

UNIT V

Z – TRANSFORMS AND DIFFFERENCE EQUATIONS

PART A

1. Find $Z\{n\}$
2. Express $Z\{f(n+1)\}$ in terms of $\bar{f}(z)$
3. Find the value of $Z\{f(n)\}$ when $f(n)=na^n$
4. Find the z-transform of n.
5. Find $Z\left\{\frac{a^n}{n!}\right\}$ using z-transform.
6. Find the z-transform of $(n+1)(n+2)$
7. Find $Z\{e^{-iat}\}$ using z-transform.

8. Find the Z-transforms of $\cos n\theta$
9. Find $Z\left(t^2 e^{-t}\right)$
10. Find $Z\{n+2\}$
12. State and prove initial value theorem in z-transform.
13. Form the difference equation from $y_n = a + b3^n$
14. Find Z-transform of $\left(\frac{1}{n}\right)$
15. Find $Z(e^t \sin 2t)$
16. Evaluate $Z^{-1}\left(\frac{z}{z^2+7z+10}\right)$
17. Prove that $Z(a^n) = \left\{\frac{z}{z-a}\right\}$
18. Find $Z\left\{\frac{1}{n(n+1)}\right\}$
19. Find $Z(n) = \frac{z}{(z-1)^2}$

PART B

1. (i)Prove that $Z\left(\frac{1}{n}\right) = \log\left(\frac{z}{z-1}\right), \quad n \neq 0$. (4)

   (ii) Find $Z\left(\sin\frac{n\pi}{2}\right)$ *and* $Z\left(\cos\frac{n\pi}{2}\right)$ (12)

2. (i)Prove that $Z\left(\frac{1}{n+1}\right) = z\log\left(\frac{z}{z-1}\right)$, (8)

   (ii) Find $Z(\sin at)$ *and* $Z(\cos at)$ (8)

3.(i) Find $Z\left(an^2+bn+c\right)$ *and* $z\{(n+1)(n+2)\}$ (10)

   (ii)Find $Z^{-1}\left\{\frac{z^3}{(z-1)^2(z-2)}\right\}$, by the method of partial fractions. (6)

4. (i) Find $Z^{-1}\left\{\frac{4z^2-2z}{z^3-5z^2+8z-4}\right\}$, by the method of partial fractions. (8)

   (ii) Find $Z^{-1}\left\{\frac{2z^2+4z}{(z-2)^3}\right\}$, by using Residue theorem. (8)

5.(i) Find $Z^{-1}\left\{\frac{z^2-3z}{(z-5)(z+2)}\right\}$, by using Residue theorem. (8)

(ii) Solve the difference equation

$y(n+3)-3y(n+1)+2y(n)=0 \quad given\ y(0)=4, y(1)=0\ and\ y(2)=8.$ (8)

6.(i) Solve the difference equation (8)

$y(n)+3y(n-1)-4y(n-2)=0, n\geq 2, \quad given\ y(0)=3, y(1)=-2$

(ii) Find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z+1)}\right)$ using Convolution theorem. (8)

7.(i) Solve the difference equation (8)

$y(k+2)-4y(k+1)+4y(k)=0 \quad given\ y(0)=1, y(1)=0$

(ii) Solve the equation $x_{n+2}-5x_{n+1}+6x_n=36$, given that $x_0=x_1=0$. (8)

8.(i)Using Convolution theorem find $Z^{-1}\left(\frac{z^2}{(z+2)^2}\right)$ (8)

(ii) Solve $u_{n+2}-5u_{n+1}+6u_n=4^n$, given that $u_0=0\ and\ u_1=1$. (8)

----------- *All the Best* ------------